

### ABSTRACT

This present paper concerns with the estimation of population mean of the study variable by utilizing the known median of the study variable. A generalized ratio type estimator has been proposed for this purpose. The expressions for the bias and mean squared error of the proposed estimator have been derived up to the first order of approximation. The optimum value of the characterizing scalar has also been obtained. The minimum value of the proposed estimator for this optimum value of the characterizing scalar is obtained. A theoretical efficiency comparison of the proposed estimator has been made with the mean per unit estimator, usual ratio estimator of Cochran (1940), usual regression estimator of Watson (1937), Bahl and Tuteja (1991), Kadilar (2016) and Subramani (2016) estimators. Through the numerical study, the theoretical findings are validated and it has been found that proposed estimate or performs better than the existing estimators.

**KEYWORDS:** Study variable, Bias, Ratio estimator, Mean squared error, Simple random sampling, Efficiency.

### I. INTRODUCTION

In many practical situations we deal with the cases where population mean of the study variable is not known but the population median of the study variable is known in advance. For example if we ask for the weight or basic salary of a person, it is very hard to get the exact value but we get the information in terms of interval or the pay band. Here we can easily get the median of the study variable which can be utilized for improved estimation of population mean of study variable. The auxiliary variable which is highly correlated with study variable is also used for improving the efficiency of the estimator but the collection of auxiliary information will increase the cost of survey. Use of median of study variable is an important attempt in this aspect. In the present paper we have developed an improved estimator of population mean of the study variable using median of the study variable.

Let us consider a finite population of  $N$  distinct and identifiable units and let  $(x_i, y_i), i = 1, 2, \dots, n$  be a bivariate sample of size  $n$  taken from  $(X, Y)$  using a simple random sampling without replacement (SRSWOR) scheme. Let  $\bar{X}$  and  $\bar{Y}$  be the population means of the auxiliary and the study variables respectively, and let  $\bar{x}$  and  $\bar{y}$  be the corresponding sample means. In SRSWOR, It is well established that sample means  $\bar{x}$  and  $\bar{y}$  are unbiased estimators of population means of  $\bar{X}$  and  $\bar{Y}$  respectively.

The above problem can be elaborated in more efficient way through an interesting example of mean estimation of study variable using median of study variable given by Subramani (2016). The table has been used with the permission of author.

**Example.** The estimation of body mass index (BMI) of the 350 patients in a Hospital based on a small simple random sample without replacement has been considered.

**Table1: Body mass index of 350 patients in a hospital**

Category	BMI range – kg/m2	Number of patients	Cumulative total
Very severely underweight	less than 15	15	15
Severely underweight	from 15.0 to 16.0	35	50
Underweight	from 16.0 to 18.5	67	117
Normal (healthy weight)	from 18.5 to 25	92	209
Overweight	from 25 to 30	47	256
Obese Class I (Moderately obese)	from 30 to 35	52	308
Obese Class II (Severely obese)	from 35 to 40	27	335
Obese Class III (Very severely obese)	over 40	15	350
Total		350	350

The median value will be between 18.5 and 25. So one can assume that the population median of the BMI is approximately 21.75.

## II. REVIEW OF EXISTING ESTIMATORS

The sample mean is the most suitable estimator of population mean of the study variable, given by,

$$t_o = \bar{y} = \frac{1}{n} \sum_{i=1}^n y_i$$

(1)

It is an unbiased estimator and its variance, up to the first order of approximation, is given by

$$V(t_o) = \frac{1-f}{n} S_y^2 = \frac{1-f}{n} \bar{Y}^2 C_y^2$$

(2)

$$\text{where, } C_y = \frac{S_y}{\bar{Y}}, S_y^2 = \frac{1}{N-1} \sum_{i=1}^N (Y_i - \bar{Y})^2 = \frac{1}{N} \sum_{i=1}^{N C_n} (\bar{y}_i - \bar{Y})^2, f = \frac{n}{N}.$$

Watson (1937) first utilized the highly correlated auxiliary variable and proposed the usual linear regression estimator of population mean as,

$$t_1 = \bar{y} + b_{yx} (\bar{X} - \bar{x})$$

(3)

Where  $b_{yx}$  is the regression coefficient of Y on X.

This estimator is also unbiased for population mean and its variance up to the first order of approximation, is given by,

$$V(t_1) = \frac{1-f}{n} \bar{Y}^2 C_y^2 (1 - \rho_{yx}^2).$$

(4)

Cochran (1940) made use of highly positively correlated auxiliary variable and proposed the following usual ratio estimator as,

$$t_2 = \bar{y} \frac{\bar{X}}{\bar{x}}$$

(5)

It is a biased estimator of population mean and the expressions for the bias and mean squared error for this estimator, up to the first order of approximation are given as,  $B(t_2) = \frac{1-f}{n} \bar{Y} [C_x^2 - C_{yx}]$  and

$$MSE(t_2) = \frac{1-f}{n} \bar{Y}^2 [C_y^2 + C_x^2 - 2C_{yx}], \tag{6}$$

where,  $C_x = \frac{S_x}{\bar{X}}$ ,  $S_x^2 = \frac{1}{N-1} \sum_{i=1}^N (X_i - \bar{X})^2 = \frac{1}{N} \sum_{i=1}^N (\bar{x}_i - \bar{X})^2$ ,  $\rho_{yx} = \frac{Cov(x, y)}{S_x S_y}$ ,

$Cov(x, y) = \frac{1}{N-1} \sum_{i=1}^N (Y_i - \bar{Y})(X_i - \bar{X})$ , and  $C_{yx} = \rho_{yx} C_y C_x$ .

Bahl and Tuteja (1991) proposed the following exponential ratio type estimator of population mean by making use of positively correlated auxiliary variable as,

$$t_3 = \bar{y} \exp\left[\frac{\bar{X} - \bar{x}}{\bar{X} + \bar{x}}\right] \tag{7}$$

The above estimator is biased and the bias and the mean squared error of this estimator, up to the first order of approximation, are given respectively by,

$$B(t_3) = \frac{1-f}{8n} \bar{Y} [3C_x^2 - 4C_{yx}] \text{ and}$$

$$MSE(t_3) = \frac{1-f}{n} \bar{Y}^2 [C_y^2 + \frac{C_x^2}{4} - C_{yx}]. \tag{8}$$

Kadilar (2016), using positively correlated auxiliary variable proposed the following exponential type estimator of population mean as,

$$t_4 = \bar{y} \left(\frac{\bar{x}}{\bar{X}}\right)^\delta \exp\left(\frac{\bar{X} - \bar{x}}{\bar{X} + \bar{x}}\right) \tag{9}$$

where  $\delta$  is a characterizing scalar to be determined such that the MSE of above estimator is minimum.

The bias and the mean squared error of the above estimator up to the first order of approximation respectively are,

$$B(t_4) = \frac{1-f}{n} \bar{Y} \left[ \left\{ \frac{\delta(\delta-1)}{2} + \frac{3}{8} \right\} C_x^2 + \left( \delta + \frac{1}{2} \right) C_{yx} \right]$$

$$MSE(t_4) = \frac{1-f}{n} \bar{Y}^2 \left[ C_y^2 + \left( \delta^2 + \delta + \frac{1}{4} \right) C_x^2 + (2\delta + 1) C_{yx} \right]$$

(10)

The optimum value of the characterizing scalar  $\delta$  which minimizes the mean squared error of  $t_6$  is,

$$\delta_{opt} = \left( \frac{1}{2} - \rho_{yx} C_y / C_x \right)$$

The minimum value of the mean squared error of above estimator is,

$$MSE_{min}(t_4) = \frac{1-f}{n} \bar{Y}^2 C_y^2 (1 - \rho_{yx}^2)$$

(11)

which is equal to the variance of the usual regression estimator of Watson (1937).

Subramani (2016) used the population median of the study variable and proposed the following ratio estimator of population mean of the study variable,

$$t_5 = \bar{y} \left( \frac{M}{m} \right)$$

(12)

where  $M$  and  $m$  are the population and sample medians of study variable respectively.

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It is a biased estimator and its bias and the mean squared error, up to the first order of approximation, are respectively given by,

$$B(t_5) = \frac{1-f}{n} \bar{Y} [C_m^2 - C_{ym} - \frac{Bias(m)}{M}] \text{ and}$$

$$MSE(t_5) = \frac{1-f}{n} \bar{Y}^2 [C_y^2 + R_5^2 C_m^2 - 2R_5 C_{ym}],$$

(13)

where,  $R_5 = \frac{\bar{Y}}{M}$ ,  $C_m = \frac{S_m}{M}$ ,  $S_m^2 = \frac{1}{N C_n} \sum_{i=1}^{N C_n} (m_i - M)^2$ ,  $S_{ym} = \frac{1}{N C_n} \sum_{i=1}^{N C_n} (\bar{y}_i - \bar{Y})(m_i - M)$  and

$$C_{ym} = \frac{S_{ym}}{\bar{Y}M}.$$

Many authors have given various modified estimators of population mean using auxiliary variables. The latest references can be found in Subramani (2013), Subramani and Kumarapandiyam (2012, 2013), Yan and Tian (2010), Yadav and Kadilar (2013), Yadav et al. (2014, 2015), and Yadav et al. (2016).

### III. PROPOSED ESTIMATOR

Motivated by Yadav and Kadilar (2013), we propose the following ratio type estimator of population mean using known population median of study variable as,

$$t_d = \bar{y} \exp \left[ \frac{M - m}{M + (a - 1)m} \right]$$

(14)

where  $a$  is a characterizing scalar to be determined such that the mean squared error of the proposed estimator  $t_d$  is minimum.

The following approximations have been made to study the properties of the proposed estimator as,

$$\bar{y} = \bar{Y}(1 + e_0) \text{ and } m = M(1 + e_1) \text{ such that } E(e_0) = 0, E(e_1) = \frac{\bar{M} - M}{M} = \frac{Bias(m)}{M} \text{ and}$$

$$E(e_0^2) = \frac{1-f}{n} C_y^2, E(e_1^2) = \frac{1-f}{n} C_m^2, E(e_0 e_1) = \frac{1-f}{n} C_{ym},$$

where,  $\bar{M} = \frac{1}{n} \sum_{i=1}^n m_i$

The proposed estimator  $t_d$  can be expressed in terms of  $e_i$ 's ( $i = 1, 2$ ) as,

$$= \bar{Y}(1 + e_0) \exp \left[ \frac{M - M(1 + e_1)}{M + (a - 1)M(1 + e_1)} \right]$$

$$= \bar{Y}(1 + e_0) \exp \left[ \frac{-e_1}{a \left\{ 1 + \frac{(a-1)}{a} e_1 \right\}} \right]$$

$$= \bar{Y}(1 + e_0) \exp \left[ -\frac{e_1}{a} \{ 1 + \theta e_1 \}^{-1} \right], \text{ where } \theta = \frac{(a - 1)}{a}$$

$$= \bar{Y}(1 + e_0) \exp \left[ -\frac{e_1}{a} (1 - \theta e_1 + \theta^2 e_1^2) \right]$$

$$t_d - \bar{Y} = \bar{Y} \left[ \left( \frac{\theta}{a} + \frac{1}{8} \right) e_1^2 + e_0 - \frac{e_0 e_1}{a} - \frac{e_1}{a} \right]$$

(15)

Taking expectation on both sides and putting the values of various expectations, we get the bias of the proposed estimator  $t_p$ , up to the first order of approximation as,

$$\text{Bias}(t_d) = \bar{Y} \left[ \left( \frac{\theta}{a} + \frac{1}{8} \right) \lambda C_m^2 - \frac{\lambda C_{ym}}{a} - \frac{\text{Bias}(M)}{aM} \right], \text{ where } \lambda = \frac{1-f}{n}$$

Squaring equation (15) both sides of above equation and taking expectations on both sides, we get the approximate mean squared error of the proposed estimator as  $t_d$  as,

$$\begin{aligned} \text{MSE}(t_d) &= \bar{Y}^2 E \left[ e_0 - \frac{e_1}{a} \right]^2 \\ &= \bar{Y}^2 E \left[ e_0^2 + \frac{e_1^2}{a^2} - \frac{2}{a} e_0 e_1 \right] \\ &= \bar{Y}^2 \left[ E(e_0^2) + \frac{E(e_1^2)}{a^2} - \frac{2}{a} E(e_0 e_1) \right] \end{aligned}$$

Putting values of various expectations in above equation, we have

$$\text{MSE}(t_d) = \lambda \bar{Y}^2 \left[ C_y^2 + \frac{C_m^2}{a^2} - \frac{2}{a} C_{ym} \right]$$

(16)

which is minimum for,

$$a_{opt} = \frac{C_m^2}{C_{ym}}$$

and the minimum mean squared error of the proposed estimator  $t_p$  is,

$$\text{MSE}(t_d)_{min} = \lambda \bar{Y}^2 \left[ C_y^2 - \frac{C_{ym}^2}{C_m^2} \right]$$

(17)

#### IV. EFFICIENCY COMPARISON

Under this section, a theoretical comparison of the proposed estimator has been made with the competing estimators of population mean. The conditions under which the proposed estimator performs better than the competing estimators have also been given.

From equation (17) and equation (2), we have,

$$V(t_0) - \text{MSE}_{min}(t_d) > 0 \text{ if } \frac{C_{ym}^2}{C_m^2} > 0, \text{ or if } C_{ym}^2 > 0$$

Thus the proposed estimator is better than the usual mean per unit estimator of population mean.

From equation (17) and equation (4), we have,

$$\text{MSE}(t_1) - \text{MSE}_{min}(t_d) > 0 \text{ if } \frac{C_{ym}^2}{C_m^2} - C_y^2 \rho_{yx}^2 > 0$$

Under the above condition, the proposed estimator is better than the usual regression estimator of Watson (1937).

From equation (17) and equation (6), we have,

$$\text{MSE}(t_2) - \text{MSE}_{min}(t_d) > 0 \text{ if } C_x^2 + \frac{C_{ym}^2}{C_m^2} > 2C_{yx}$$

Under the above condition, proposed estimators perform better than the usual ratio estimator given by Cochran (1940).

From equation (17) and equation (8), we have,

$$MSE(t_3) - MSE_{\min}(t_d) > 0 \text{ if } \frac{C_x^2}{4} + \frac{C_{ym}^2}{C_m^2} > C_{yx}$$

Under the above condition, the proposed estimator performs better than Bahl and Tuteja (1991) ratio type estimator of population mean.

From equation (17) and equation (11), we have,

$$MSE(t_4) - MSE_{\min}(t_d) > 0, \text{ if } \frac{C_{ym}^2}{C_m^2} - C_y^2 \rho_{yx}^2 > 0$$

Under the above condition the proposed estimator is better than Kadilar (2016) estimator of population mean using auxiliary information.

From equation (17) and equation (13), we have,

$$MSE(t_5) - MSE_{\min}(t_d) > 0, \text{ if } R_5^2 C_m^2 + \frac{C_{ym}^2}{C_m^2} > 2R_5 C_{ym}$$

Under the above condition, the proposed estimator is better than the Subramani (2016) estimator of population mean using information on median of the study variable.

### V. NUMERICAL STUDY

To judge the theoretical findings, we have considered the natural populations given in Subramani (2016). He has used three natural populations. The population 1 and 2 have been taken from Singh and Chaudhary (1986, page no. 177) and the population 3 has been taken from Mukhopadhyay (2005, page no. 96). In populations 1 and 2, the study variable is the estimate the area of cultivation under wheat in the year 1974, whereas the auxiliary variables are the cultivated areas under wheat in 1971 and 1973 respectively. In population 3, the study variable is the quantity of raw materials in lakhs of bales and the number of labourers as the auxiliary variable, in thousand for 20 jute mills. Tables 3 and 4 represent the parameter values along with constants, along with proposed estimator, variances and mean squared errors of existing and proposed estimator

**Table-3. Parameter values and constants for three natural populations**

Parameter	Population-1	Population-2	Population-3
$N$	34	34	20
$n$	5	5	5
${}^N C_n$	278256	278256	15504
$\bar{Y}$	856.4118	856.4118	41.5
$\bar{M}$	736.9811	736.9811	40.0552
$M$	767.5	767.5	40.5
$\bar{X}$	208.8824	199.4412	441.95
$R_7$	1.1158	1.1158	1.0247
$C_y^2$	0.125014	0.125014	0.008338
$C_x^2$	0.088563	0.096771	0.007845
$C_m^2$	0.100833	0.100833	0.006606
$C_{ym}$	0.07314	0.07314	0.005394
$C_{yx}$	0.047257	0.048981	0.005275
$\rho_{yx}$	0.4491	0.4453	0.6522

*Table-4. Mean squared error of various estimators*

Estimator	Popln-1	Popln-2	Popln-3
$t_0$	15640.97	15640.97	2.15
$t_1$	12486.75	12539.30	1.24
$t_2$	14895.27	15492.08	1.48
$t_3$	12498.01	12539.30	1.30
$t_4$	12486.75	12539.30	1.24
$t_5$	10926.53	10926.53	1.09
$t_d$	9002.22	9002.22	0.98

## VI. RESULTS AND CONCLUSION

From Table-4, it can be seen that the proposed estimator has minimum mean squared error among other competing estimators of population mean of study character. Thus proposed estimator is better than usual mean per unit estimator, Watson (1937) usual regression estimator, Cochran (1940) usual ratio estimator, Bahl and Tuteja (1991) exponential ratio type estimator, Kadilar (2016) estimator and Subramani (2016) estimator. Therefore it is recommended that the proposed estimator may be used for improved estimation of population mean under simple random sampling scheme.

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